MATH266: First midterm test, February 10th

Name: _____

Write you name on the title page of the bluebook. Start each problem on a new page. Provide arguments. Present your solutions in a logical fashion. Make sure that you answer all the posed questions. Emphasize your answers.

1. Is the following equation separable? It is linear? It is exact?

$$x^2y' = y - xy$$

   Solve this equation with the initial condition $y(-1) = -1$.

2. Solve the IVP

$$(e^x + y)dx + (2 + x + ye^y)dy = 0, \quad y(0) = 1.$$

3. Treat the following logistic equation as Bernoulli's equation and find its general solution:

$$y' = 3y(1 - y).$$

   What is the limit of $y(t)$ when $t \to \infty$ if (a) $y(0) = 0.0001$, (b) $y(0) = -0.0001$?

4. Formulate Newton's law of cooling and write down the corresponding ODE.

   A small metal bar, whose initial temperature was 20°C, is dropped into a large container of boiling water (water boils at 100°C). How long will it take the bar to reach 90°C if it is known that its temperature increases 2°C during the first second (to obtain the answer you can use $\log 8 \approx 2.1$, $\log(40/39) \approx 0.025$).

5. Find and classify equilibria of the autonomous differential equation

$$\dot{x} = 4x - x^3.$$

   Draw the phase portrait and sketch several representative integral curves of this equation. Is it possible to have an initial condition $x(0) = x_0$ such that the solution with this condition will tend to $\pm\infty$ as $t \to \infty$? If the answer is positive, find all such x_0.